The Adjoint School 2021

Example Syllabus

Contents

1 Overview

During the online seminar each of the 8 papers will be co-lead by a team of 2-3 students. This team is responsible for

- giving a presentation (see Section 4)
- writing a blog post (see Section 5)

In addition, each individual participant is responsible for attending every presentation and posting a reading response for each paper. See Section 3 for details.

During the research week (July 5 - 9) students will work on a research project under the guidance of their project's mentor. You should expect to work with your group for 5 full workdays.

We will use Zulip (see section 7) as our main mode of communication. It will be used for general announcements, posting reading responses, and informal conversations.

2 Schedule

In the schedule below, each paper is given a two week block which is roughly divided into a reading week and a presentation week. In the first week all participants will be reading the assigned paper and submitting a reading response (see Section 3). In the second week, the pair in charge of the reading will give a presentation. The exact meeting times during the presentation weeks will be scheduled during the initial meeting.

Feb 24	Introductions and kickoff meeting
March 1-9	*Universal coalgebra* - reading Sections 1-3 and 8-13
March 10	*Universal coalgebra* - presentation by group 4.2
March 15-23	*On the fuzzy concept complex* - reading Chapters 2 and 3
March 24	*On the fuzzy concept complex* - presentation by group 3.2
March 29 - April 6	*Decorated Cospans* - reading
April 7	*Decorated Cospans* - presentation by group 1.1
April 12 - 20	*Structured Cospans* - reading
April 21	*Structured Cospans* - presentation by group 2.1
April 26 - May 4	*Algebras of Open Dynamical Systems* - reading
May 5	*Algebras of Open Dynamical Systems* - presentation by group 1.2
May 10-18	*The Legendre-Fenchel transform* - reading
May 19	*The Legendre-Fenchel transform* - presentation by group 3.1
May 24 - June 1	*Coalgebraic semantics of modal logics* - reading
June 2	*Coalgebraic semantics of modal logics* - presentation by group 4.1
June 14-22	*Weak 2-categories* - reading
June 23	*Weak 2-categories* - presentation by group 2.2
June 28 - July 2	Break
July 5-9	Adjoint School Research Week
July 12-16	Applied Category Theory 2021 conference

This schedule can also be found on the school's Google calendar, which will be shared with participants.

3 Reading Responses

Reading responses are a critical part of this school. Every student is responsible for writing a reading response for every paper. These need not be long (think a paragraph).

What is the purpose of these reading responses? Engaging with new papers is often difficult, confusing, and frustrating, and a lot can be learned, about math and doing math research, by sharing this experience in a group. Reading responses help spark these discussions.

If you are stumped for what to write, set a timer for 15 minutes and answer one of the following prompts:

- What was it like for you to read the paper? Fun? Frustrating?
- What are the big ideas in this paper?
- What are the big ideas in this paper? But this time like you are explaining it to yourself in middle school.
- What was the most interesting idea to you?
- Did it prompt you to explore any new directions or background material? Tell us about that.
- Do you aspire to write like the authors? What is one thing you would (or would not) want to emulate?

The reading responses are due the day prior to the associated presentation. We encourage you to respond to each other's reading responses.

4 Presentations

Each group will make two presentations, one per paper. For those groups with additional readings, the content labeled "extra" is provided as an additional resource to strengthen your background knowledge. Section 8 lists the paper you were assigned to present and write about. You will have a partner. Each partnership may determine how to split the workload. We encourage collaboration above a disjoint workflow.

Each presentation should be 40 minutes. Please take care to adhere to this timing as a courtesy for all participants. With the rest of our meeting time, we will have 15 minutes for small group discussion and 20 minutes for large group conversation. All in all, the meeting is expected to last roughly at 1 hour and 20 minutes.

Please post any slides or notes to the school Zulip prior to your presentation. Technical details are in Section 7.

5 Blog Posts

Each team will write a blog post inspired by the paper they read. This does not need to be a summary of the paper. Indeed, try to incorporate the project and related papers in this school. Both pair of partners will receive authorship of the paper, regardless of who does the majority writing.

We expect this article to be of professional quality. Thus, it will go through several rounds of revisions. After you complete your first draft, allow your TA and project lead to look at it. Also, allow the organizers Sophie and David to look. Incorporate any notes you are given.

Because it will be posted as a blog article, there is no maximum length. However, we do want busy readers to make it to the end of the article, so keep the readers attention span in mind.

The article will be posted on the nCafe.

6 Video Conferencing

We will use Zoom for our whole school meetings.

7 Zulip

We will be using an online message board Zulip for our discussions during the program. You will be sent an invite link to join the school's Zulip.

8 Project Descriptions

How to read the groups: If you are in group 3 and assigned reading 1, then you are called group 3.1 in the schedule (see Section 2).

Group 1

Title. Categorical and computational aspects of C-sets

Members.

- James Fairbanks (Mentor)
- Evan Patterson (Mentor)
- Christian Willimas (TA)
- Owen Lynch (TA)
- Angeline Aguinaldo (reading 1)
- Anna Knörr (reading 1)
- Grant Generaux (reading 1)
- Amy Searle (reading 2)
- Kris Brown (reading 2)
- Marco Perin (reading 2)

Description.

Applied category theory includes major threads of inquiry into monoidal categories and hypergraph categories for describing systems in terms of processes or networks of interacting components. Structured cospans are an important class of hypergraph categories. For example, Petri net-structured cospans are models of concurrent processes in chemistry, epidemiology, and computer science. When the structured cospans are given by C-sets (also known as co-presheaves), generic software can be implemented using the mathematics of functor categories. We will study mathematical and computational aspects of these categorical constructions, as well as applications to scientific computing.

Reading.

1. Fong, *Decorated Cospans.* Available at

   https://arxiv.org/abs/1502.00872

2. Wagner, Spivak, Lerman, *Algebras of Open Dynamical Systems on the Operad of Wiring Diagrams.* Available at

   https://arxiv.org/abs/1408.1598

Group 2

Title. Double categories in applied category theory

Members.

- Simona Paoli (Mentor)
- Maru Sarazola (TA)
- Daniel Plácido (reading 1)
- Elise McMahon (reading 1)
- Claire Ott (reading 2)
- Emma Phillips (reading 2)

Description. Bicategories and double categories (and their symmetric monoidal versions) have recently featured in applied category theory: for instance, structured cospans and decorated cospans have been used to model several examples, such as electric circuits, Petri nets and chemical reaction networks. An approach to bicategories and double categories is available in higher category theory through models that do not require a direct checking of the coherence axioms, such as the Segal-type models. We aim to revisit the structures used in applications in the light of these approaches, in the hope to facilitate the construction of new examples of interest in applications.

Reading.

1. Baez and Courser, *Structured Cospans*. Available at

   https://arxiv.org/abs/1911.04630

2. Paoli and Pronk, *A Double Categorical Model of Weak 2-categories*. Available at

   http://www.tac.mta.ca/tac//volumes/28/27/28-27.pdf

3. (extra) Paoli, *Simplicial Methods for Higher Categories*.

Group 3

Title. The ubiquity of enriched profunctor nuclei

Members.

- Simon Willerton (Mentor)
- Tai-Danae Bradley (TA)
- Elise Catania (reading 1)
- Hussain Kadhem (reading 1)
- Matt Di Meglio (reading 2)
- Owen Lewis (reading 2)

Description. In 1964, Isbell developed a nice universal embedding for metric spaces: the tight span. In 1966, Isbell developed a duality for presheaves. These are both closely related to enriched profunctor nuclei, but the connection wasn't spotted for 40 years. Since then, many constructions in mathematics have been observed to be enriched profunctor nuclei too, such as the fuzzy/formal concept lattice, tropical convex hull, and the Legendre-Fenchel transform. We'll explore the world of enriched profunctor nuclei, perhaps seeking out further useful examples.

Reading.

1. Willerton, *The Legendre-Fenchel transform from a category theoretic perspective*. Available at

   https://arxiv.org/abs/1501.03791

2. Elliott, *On the fuzzy concept complex*, (Chapters 2-3). Available at

   https://etheses.whiterose.ac.uk/18342/1/JAE%20thesis%20%28White%20Rose%29.pdf

Group 4

Title. Extensions of coalgebraic dynamic logic

Members.

- Helle Hvid Hansen (Mentor)
- Clemens Kupke (Mentor)
- Toby St. Clere Smithe (TA)
- Gabriel Goren (reading 1)
- Anthony Agwu (reading 1)
- Amin Karamlou (reading 2)
- Stelios Tsampas (reading 2)

Description. Coalgebra is a branch of category theory in which different types of state-based systems are studied in a uniform framework, parametric in an endofunctor $F : \mathcal{C} \to \mathcal{C}$ that specifies the system type. Many of the systems that arise in computer science, including deterministic/nondeterministic/weighted/probabilistic automata, labelled transition systems, Markov chains, Kripke models and neighbourhood structures, can be modeled as F-coalgebras. Once we recognise that a class of systems are coalgebras, we obtain general coalgebraic notions of morphism, bisimulation, coinduction and observable behaviour.

Modal logics are well-known formalisms for specifying properties of state-based systems, and one of the central contributions of coalgebra has been to show that modal logics for coalgebras can be developed in the general parametric setting, and many results can be proved at the abstract level of coalgebras. This area is called coalgebraic modal logic.

In this project, we will focus on coalgebraic dynamic logic, a coalgebraic framework that encompasses Propositional Dynamic Logic (PDL) and Parikh's Game Logic. The aim is to extend coalgebraic dynamic logic to system types with probabilities. As a concrete starting point, we aim to give a coalgebraic account of stochastic game logic, and apply the coalgebraic framework to prove new expressiveness and completeness results.

Participants in this project would ideally have some prior knowledge of modal logic and PDL, as well as some familiarity with monads.

Reading.

1. Rutten, *Universal coalgebra: A theory of systems* (Sections 1-3, 9-13). Available at

   https://www.sciencedirect.com/science/article/pii/S0304397500000566

2. Kupke and Pattinson, *Coalgebraic semantics of modal logics: An overview*. Available at

   https://www.sciencedirect.com/science/article/pii/S0304397511003215

3. (extra) Hansen, Kupke, and Leale, *Strong completeness of iteration-free coalgebraic dynamic logics*. Available at

   https://hal.inria.fr/hal-01402072/document